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Mathematical analysis for the efficiency of a semi-spherical fin with simultaneous heat and mass transfer

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ABSTRACT

In this paper an analytical solution for the efficiency of a semi-spherical fin when subjected to simultaneous heat and mass transfer mechanisms is studied. For the mathematical analysis of a wet fin equation, a relationship between humidity ratio and temperature of the saturation air is needed. The driving forces for the heat and mass transfer are the temperature and humidity ratio differences, respectively. Analytical solutions are obtained for the temperature distribution over the fin surface when the fin is fully wet. It is observed that in humid conditions the fin has high efficiency to be used in industry. The variation effects of these parameters have been considered. Finally linear relation has been proposed for humidity and temperature on the fin surface.

Analyse mathématique de l'efficacité d'une ailette semi-sphérique avec transfert de chaleur et de masse simultanés

Mots clés : Cilette ; Semi-sphérique ; Efficacité ; Transfert de chaleur ; Transfert de masse ; Simulation

1. Introduction

Extended surfaces that are well known as fins are commonly used to enhance heat transfer in many industries (Shaeri et al., 2009). Fins are widely used to increase the rate of heat transfer from a primary wall surface. An increase in heat transfer rate is due to reduction in surface resistance. Depending upon the base temperature in comparison to the surrounding temperature decides the direction of heat transfer. For an example, heat transfer takes place by dissipation of the transfer process due to maintaining a higher fin surface temperature in comparison to the surrounding. There is lot of practical applications in which the base temperature is kept at a lower value than the surrounding temperature. In these

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Nomenclature

A (m^2) fin cross-sectional area
a_2 ($kg_w\,kg_a^{-1}$) constants defined in Eq. (12)
B (C) parameter defined in Eq. (8)
b_2 ($kg_w\,kg_a^{-1}\,K^{-1}$) constants defined in Eq. (13)
C_0 constant defined in Eq. (16)
C constant defined in Eq. (10)
c_p ($J\,kg^{-1}\,K^{-1}$) specific heat of air
H_a (%) percentage humidity
h ($W\,m^{-2}\,K^{-1}$) heat transfer coefficient on the air side
h_D ($kg\,s^{-1}\,m^{-2}\,kg_w\,kg_a^{-1}$) convective mass transfer coefficient
i_{fg} ($J\,kg^{-1}$) latent heat of evaporation of water
k ($W\,m^{-1}\,K^{-1}$) thermal conductivity of the fin material
Le Lewis number
m (m^{-1}) wet fin parameter defined in Eq. (15)
m_0 (m^{-1}) dry fin parameter defined in Eq. (7)
P_{atm} (atm) atmospheric pressure
q (W) heat transfer rate
R (m) semi-spherical fin radius
T (°C) temperature
x (m) radius

Greek letters
η fin efficiency
θ (°C) temperature difference
ω ($kg_w\,kg_a^{-1}$) humidity ratio of air

Superscripts and subscripts
a air
b fin base
dp dew point
s fin surface
w water

components, the heat is transferred from the surrounding to the fin surface. In refrigeration and air conditioning systems, the air being cooled when it passes over the fin surface maintained at a lower temperature. If the fin surface temperature is below the dew-point temperature of the surrounding air, the simultaneous heat and mass transfer mechanism takes place on the fin surface. The humid air strikes on a surface which is below the dew-point temperature, condensation of moisture occurs on the fin surface by evolving the latent heat of condensation. The motive force, temperature difference between surrounding and fin surface, is responsible for sensible heat transfer and difference of humidity ratio between the surrounding and that on the fin surface is the motive force for the mass transfer. The analysis of fins with considering mass transfer is difficult due to complex phenomena involved (Kundu, 2010). Various types of heat-exchanger fins, ranging from relatively simple shapes, such as rectangular, cylindrical, annular, tapered or pin fins, to a combination of different geometries, have been used (Yakut et al., 2006). The selection of any particular type of fin depends on the geometry of a primary surface. The profile of a fin is generally chosen on the basis of the cost of material and manufacturing as well as on the ease of fabrication. Finned semi-spherical heat exchangers are widely used in many industrial applications such as, aerospace, electronic kits, and chemical processing systems. In cooling and dehumidification processes heat and mass transfer occurs simultaneously when the coil surface temperature is below the dew-point temperature of the air being cooled. If the fin surface temperature is higher than the environment dew point, only sensible heat transfers from air to the fin and fin is fully dry. If temperature of total fin surface is below the dew point and both of sensible and latent heat generate, fin would be fully wet. Fin is partially wet when the fin base temperature is below the air dew point and the fin tip temperature is higher than the air dew point. Study on efficiency of fins in different conditions such as fully dry, partially wet and fully wet for the straight fins is done while circular fins were not considered as much. In this study efficiency of semi-spherical fin in fully wet conditions has been considered. Several attempts have been made to analyze the fin efficiency with condensation from moist air, but the studies almost carried out about straight fins and in limited cases have been investigated circular fins under the fully wet condition, but efficiency's study of semi-spherical fin under fully wet condition is carried out for first time in this paper.

Sharqawy and Zubair (2007) considered the efficiency of an annular fin in simultaneous mass and heat transfer situations. They obtained an analytical solution for efficiency and rate of heat transfer for an annular fin when exposed to fully wet condition. McQuiston (1975) studied analytically the overall efficiency of a fully wet straight fin. He assumed that the driving force for the mass transfer, as given by the difference in the humidity ratio between the incoming air and the existing one on the fin surface, is linearly related to the corresponding temperature difference. An analytical solution for the fin efficiency similar to that of the fin efficiency with only heat transfer (no mass transfer) is obtained. He demonstrated that the overall fin efficiency depends strongly on the relative humidity of the incoming air stream.

Elmahdy and Briggs (1983) studied numerically the overall fin efficiency of a circular fully wet fin. They assumed a linear relationship between the humidity ratio of the saturated air on the fin surface and its temperature, which is somewhat different than McQuiston's model (McQuiston, 1975). Numerical solutions for a specific circular fin were obtained. Their results indicate that the fin efficiency strongly depends on the relative humidity.

Wu and Bong (1994) provided an analytical solution for the efficiency of a straight fin under both fully wet and partially wet conditions by using the temperature and humidity ratio differences as the driving forces for heat and mass transfer. He assumed the same linear relationship between the humidity ratio of the saturated air on the fin surface and its temperature as that of Elmahdy and Briggs (1983). Their result shows that there is not much change in the fin efficiency with the relative humidity. Sharqawy and Zubair (2008) provided an analytical solution for the efficiency of a straight fin under fully wet conditions by using the temperature and humidity ratio differences as the driving forces for heat and mass transfer mechanisms, respectively. They assumed a new linear model for the relationship between the humidity ratio

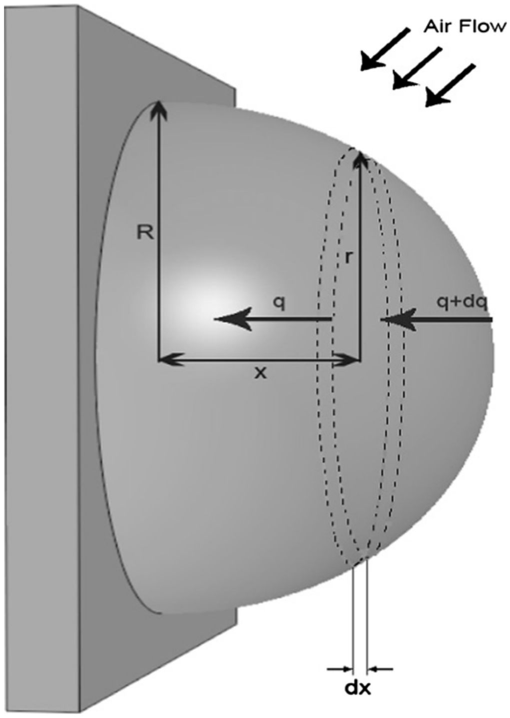

Fig. 1 – Schematic of a fully wet semi-spherical fin.

and temperature with some differences with Elmahdy and Briggs (1983) and Wu and Bong (1994) model.

The objective of this paper is to provide an analytical solution for the efficiency of a semi-spherical fin under fully wet conditions by using the temperature and humidity ratio differences as the driving forces for heat and mass transfer mechanisms, respectively. A same linear model with Sharqawy and Zubair (2008) model is assumed for the relationship between the humidity ratio and temperature.

2. Mathematical model

A steady state analysis is carried out on an annular fin when exposed to moving moist air stream, as shown in Fig. 1. In this regard, the following assumptions are made to simplify the analysis. (1) The thermal conductivity of the fin, heat transfer coefficient and latent heat of condensation of the water vapor are constant; (2) The moist air flow is steady with uniform velocities; (3) The thermal resistance associated with the presence of thin water film due to condensation is small and may be neglected; (4) The effect of air pressure drop due to air flow is neglected. These are essentially the classical assumptions that are typically used for the analysis of conducting–convecting finned surfaces.

Applying an energy balance on an infinitesimal area, $dA = 2\pi r dx$, which is shown in Fig. 1.

$$(q + dq) + 2\pi r h (T_a - T_s) dr + 2\pi r h_D i_{fg} (\omega_a - \omega_s) dr - q = 0 \tag{1}$$

According to Fig. 1 the relation between two parameters r and x can expressed by:

$$r = \sqrt{(R^2 - x^2)} \tag{2}$$

and

$$dA = 2\pi \sqrt{(R^2 - x^2)} dx \tag{3}$$

From Fourier law of heat conduction:

$$q = \pi r^2 k \frac{dT_s}{dr} \tag{4}$$

The heat transfer and mass transfer coefficients are related by the Chiltone–Colburn analogy (Threlkeld, 1970).

$$\frac{h}{h_D} = c_p Le^{2/3} \tag{5}$$

Therefore the energy balance on the elemental volume yields the following differential equation:

$$(R^2 - x^2)\frac{d^2\theta}{dx^2} - 2x\frac{d\theta}{dx} - m_0^2\sqrt{(R^2 - x^2)}\theta - m_0^2 B\sqrt{(R^2 - x^2)}(\omega_a - \omega_s) = 0 \tag{6}$$

where

$$m_0 = \sqrt{\frac{2h}{kR}} \tag{7}$$

$$B = \frac{i_{fg}}{c_p Le^{2/3}} \tag{8}$$

$$\theta = T_a - T_s \tag{9}$$

To solve Eq. (6), an additional equation is required. McQuiston (1975) assumed that:

$$\omega_a - \omega_s = C(T_a - T_s) \tag{10}$$

where C is a constant, but this equation is not a general physical relationship. However, Sharqawy and Zubair (2008) used a linear relationship between ω_s and T_s over the temperature range ($T_b < T_s < T_{dp}$)

$$\omega_s = a_2 + b_2 T_s \tag{11}$$

where

$$a_2 = \omega_{s,b} - \frac{\omega_{s,dp} - \omega_{s,b}}{T_{dp} - T_b} T_b \tag{12}$$

$$b_2 = \frac{\omega_{s,t} - \omega_{s,b}}{T_{dp} - T_b} \tag{13}$$

While this assumption looks physically acceptable, it is used in this work. Substituting Eq. (11) into Eq. (6):

$$\left(\frac{(R^2 - x^2)}{R}\right)\frac{d^2\theta}{dx^2} - (2x/R)\frac{d\theta}{dx} - m^2\sqrt{(R^2 - x^2)}\theta = m_0^2 B C_0 \sqrt{(R^2 - x^2)} \tag{14}$$

where

$$m^2 = m_0^2(1 + b_2 B) \tag{15}$$

$$C_0 = \omega_a - a_2 - b_2 T_a \tag{16}$$

Eq. (14) is a non-homogeneous second-order differential equation with the following boundary condition:

$$\theta = \theta_b; \quad x = 0 \tag{17}$$

$$\frac{d\theta}{dx} = 0; \quad x = R \tag{18}$$

The solution of Eq. (14) gives the temperature distribution along the fin surface (with power series method):

$$\theta(x) = \theta_1(x) + \theta_2(x) + \theta_3(x) \tag{19}$$

$$\theta_1(x) = \theta(0) + \theta'(0)x + \frac{1}{2}\left[m^2\theta(0) + m_0^2BC_0\right]x^2 + \frac{1}{6}\left[\theta'(0)\left[\frac{m^2R^2+2}{R^2}\right]\right]x^3 \tag{20}$$

$$\theta_2(x) = \frac{1}{24R^2}\left[5m^2\theta(0) + 5m_0^2BC_0 + m^4R^2\theta(0) + m^2R^2m_0^2BC_0\right]x^4 \tag{21}$$

$$\theta_3(x) = \frac{1}{120}\theta'(0)\left[\frac{m^4R^4 + 11m^2R^2 + 24}{R^4}\right]x^5 + O(x^6) \tag{22}$$

In this work only the $\theta_1(x)$ term is used because other terms with upper degree of x parameter are negligible when radius of the fin is small.

$$\theta(x) = \theta(0) + \theta'(0)x + \frac{1}{2}\left[m^2\theta(0) + m_0^2BC_0\right]x^2 + \frac{1}{6}\left[\theta'(0)\left[\frac{m^2R^2+2}{R^2}\right]\right]x^3 \tag{23}$$

According to Eq. (18) heat transfer from the fin tip is negligible. Eq. (23) subjected to boundary conditions (17) and (18) gives,

$$\theta(0) = \theta_b \tag{24}$$

and

$$\theta'(0) = \frac{-2\left[m^2\theta(0) + m_0^2BC_0\right]R}{m^2R^2 + 4} \tag{25}$$

The efficiency of a fin with combined heat and mass transfer under fully wet condition is expressed by Lin and Jang (2002):

$$\eta = \frac{\int\left[c_p(T_a - T_s) + \frac{1}{(Le)^{2/3}}(\omega_a - \omega_s)i_{fg}\right]dA}{\int\left[c_p(T_a - T_b) + \frac{1}{(Le)^{2/3}}(\omega_a - \omega_b)i_{fg}\right]dA} \tag{26}$$

Table 1 – The necessary data for calculating constants a_2 and b_2.

T_a (°C)	H_a (%)	ω_a ($kg_w\,kg_a^{-1}$)	T_{dp} (°C)	$\omega_{s,dp}$ ($kg_w\,kg_a^{-1}$)	T_b (°C)	$\omega_{s,b}$ ($kg_w\,kg_a^{-1}$)
20	80	0.014	17.22	0.0125	15	0.012
30	80	0.021	25.55	0.0175	15	0.012
40	80	0.040	36.11	0.033	15	0.012
50	80	0.068	46.11	0.055	15	0.012

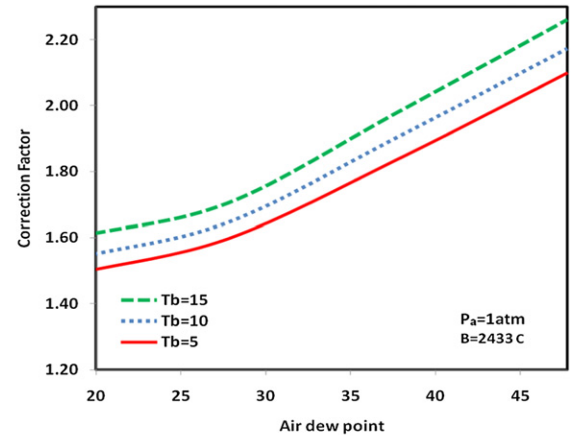

Fig. 2 – Fin parameter correction factor ($(1 + b_2B)^{1/2}$) as a function of fin base and dew-point temperatures (T_{dp}).

$$\eta = \frac{(1 + b_2B)\theta_4(x) + BC_0}{(1 + b_2B)\theta_b + BC_0} \tag{27}$$

where

$$\theta_4(x) = \theta(0) + \frac{2}{3}\theta'(0)x + \frac{1}{4}\left[m^2\theta(0) + m_0^2BC_0\right]x^2 + \frac{1}{15}\theta'(0)\left[\frac{m^2R^2+2}{R^2}\right]x^3 \tag{28}$$

The necessary data for calculating constants a_2 and b_2 by Eqs. (12) and (13) are extracted from Humidity chart (McCabe et al., 2005) and some of them are shown in Table 1:

In Table 1 is assumed that H_a and T_b are 80% and 15 °C, respectively.

3. Results and discussion

For the calculation of the fin efficiency in different conditions, the indexes of the Eq. (27) should be determined. The quantities of m_0, B, b_2 and C_0, can be calculated from Eqs. (7), (8), (13) and (16), respectively. In order to find the fin parameter in

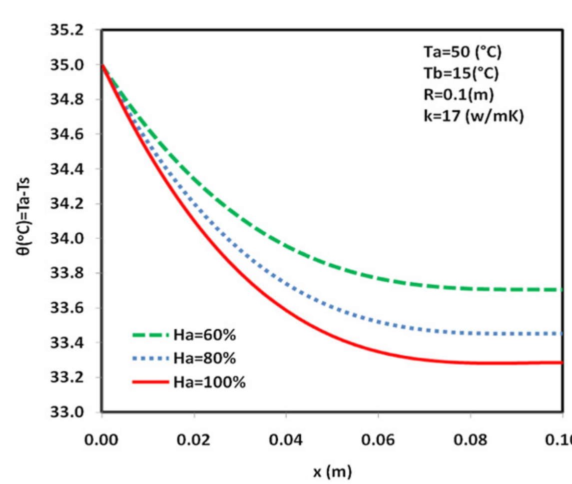

Fig. 3 – Temperature distribution at different percentage humidity.

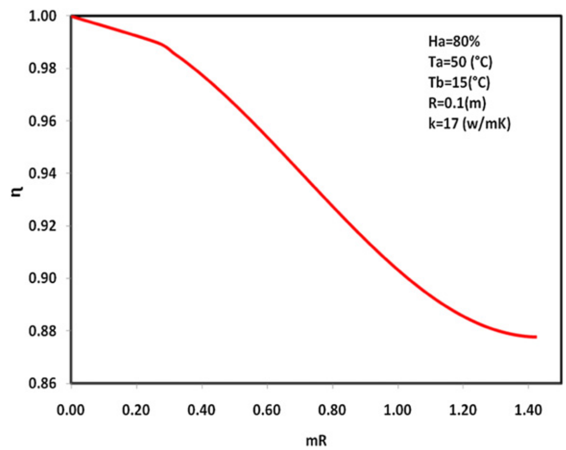

Fig. 4 – Fin efficiency versus the fin parameter (mR).

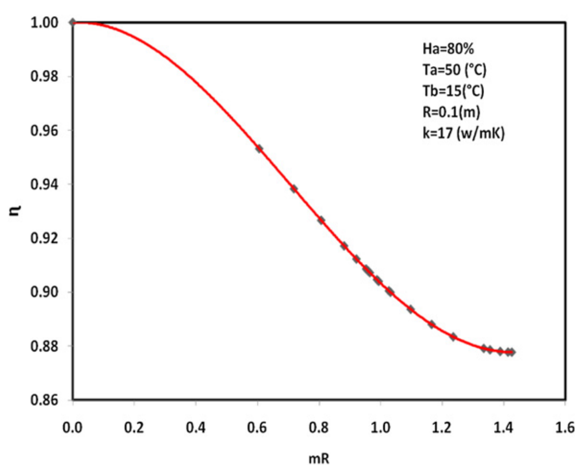

Fig. 5 – Fin efficiency versus the fin parameter with after fitting.

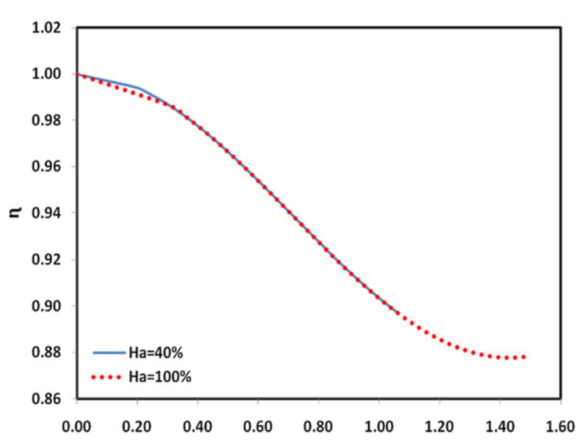

Fig. 6 – Fin efficiency versus fin parameter (mR) for 40% and 80% humidity.

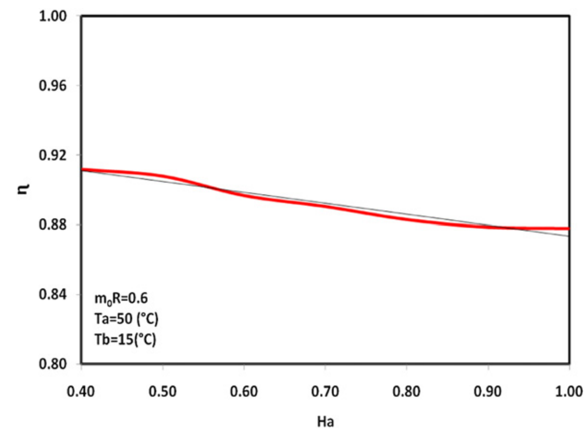

Fig. 7 – Fin efficiency for a fully wet semi-spherical fin versus percentage humidity.

different temperatures Fig. 2 is being used. With the knowledge of correction factor $(1+b_2B)^{1/2}$ and m_0 and utilization of Eq. (15), fin parameter is calculated. Using Figs. 2 and 5, can be easily find fin efficiency in fully wet condition.

Fig. 2 shows that the correction factor $(1+b_2B)^{1/2}$ is increased by the increase of T_{dp} at a constant T_b and also by the increase of T_b.

The distribution diagram of fin temperature is illustrated in Fig. 3. Of course it should be considered that this diagram is illustrated for hemispheric fin with radius of 0.1 mm and of a special material with low thermal conductivity. The air temperature is 50 °C and the base temperature of fin is 15 °C .Temperature distribution of different humidity percents is illustrated.

Temperature distribution on fin surface versus distance from fin basis is plotted for 60, 80, 100 humidity percent. For these humidity percentages, temperature of fin tip is under dew point therefore fin will be fully wet.

According to Fig. 3, if the humidity percent is increased, the difference temperature between fin and air is decreased. Thus, whatever we come closer to fully wet condition, this difference is decreased and fin temperature is increased.

Fig. 4 is illustrating the efficiency diagram of fin. This diagram is illustrated in the condition of 80 percent of humidity and it expresses efficiency at the fin base. Fin efficiency decreases with increasing of the fin parameter. Fig. 5 is illustrating the fin efficiency diagram after fitting, and the equation of this curve is mentioned in Eq. (29).

$$\eta = -0.0165(mR)^5+0.0563(mR)^4+0.0244(mR)^3-0.01632(mR)^2 +0.0023(mR)+1 \quad (29)$$

The amount of residual error is very low that conveys the little difference between the real values of efficiency and the values of efficiency after fitting.

The fin efficiency diagram at humidity percents of 40 and 100 is illustrated in Fig. 6. It is noticed that the efficiency curves are almost conforming on each other. This subject is true for other humidity percents at fully wet conditions.

Fig. 7 is illustrating the fin efficiency diagram with regard to humidity percents of 40–100, that at this range the condition is fully wet. Figs. 6 and 7 demonstrate little connection of efficiency to the humidity percent, that it is similar to Wu and Bong (1994) and Sharqawy and Zubair (2007) results.

A valuable achieved result is related to effect of geometric shape of fin on efficiency. According to reference Sharqawy and Zubair (2008) for various straight fin shapes with determining ratio of periphery to surface (periphery/area) can be concluded that in 2-dimensional shapes when this ratio (periphery/area) is decreased, fin efficiency will be increased. It is preferable that high efficiency of hemisphere fin in comparable with others geometric shapes is due to small (periphery/area) ratio.

4. Conclusions

A closed-form analytical solution has been obtained for the efficiency of a semi-spherical fin with combined heat and mass transfer. In this system the temperature and humidity ratio differences are the driving forces for the heat and mass transfer, respectively. Analytical solutions are obtained for the temperature distribution over the fin surface when the fin is fully wet.

The results show that the overall fin efficiency depends on the condition of the fin surface and has not much connection to the humidity percent. In addition, by closing to the fully wet conditions the fin temperature increases, a correction diagram according to dry fin parameter (m_0) can be plotted for determining the wet fin parameter (m) and as ratio of periphery to area (periphery/area) in fin geometric shape decreases, fin efficiency will increase and this can justify the high efficiency of semi-spherical fin.

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